5.2 Use Perpendicular Bisectors

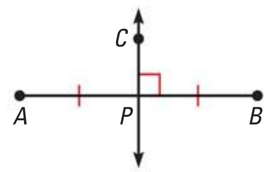
Before You used segment bisectors and perpendicular lines.

Now You will use perpendicular bisectors to solve problems.

Why? So you can solve a problem in archaeology, as in Ex. 28.

Key Vocabulary
- **perpendicular bisector**
- **equidistant**
- **concurrent**
- **point of concurrency**
- **circumcenter**

In Lesson 1.3, you learned that a segment bisector intersects a segment at its midpoint. A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a **perpendicular bisector**.



$\overleftrightarrow{CP}$ **is a $\perp$ bisector of $\overline{AB}$.**

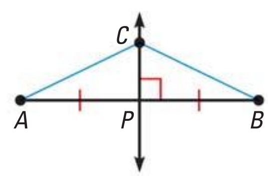
A point is **equidistant** from two figures if the point is the *same distance* from each figure. Points on the perpendicular bisector of a segment are equidistant from the segment's endpoints.

THEOREMS — *For Your Notebook*

THEOREM 5.2 Perpendicular Bisector Theorem

In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

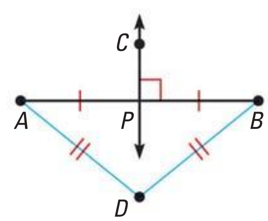


Proof: Ex. 26, p. 308

THEOREM 5.3 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

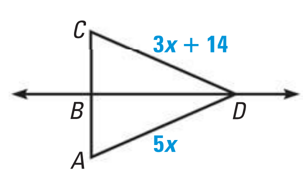
If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.



Proof: Ex. 27, p. 308

EXAMPLE 1 Use the Perpendicular Bisector Theorem

ALGEBRA **$\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. Find AD.**



$AD = CD$ — Perpendicular Bisector Theorem

$5x = 3x + 14$ — Substitute.

$x = 7$ — Solve for x.

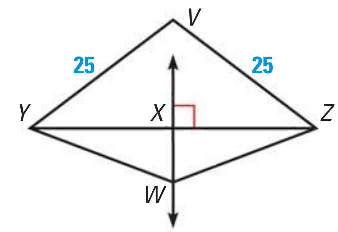
▶ $AD = 5x = 5(7) = 35$.

EXAMPLE 2 Use perpendicular bisectors

In the diagram, $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{YZ}$.

a. What segment lengths in the diagram are equal?

b. Is V on $\overleftrightarrow{WX}$?



Solution

a. $\overleftrightarrow{WX}$ bisects $\overline{YZ}$, so $XY = XZ$. Because W is on the perpendicular bisector of $\overline{YZ}$, $WY = WZ$ by Theorem 5.2. The diagram shows that $VY = VZ = 25$.

b. Because $VY = VZ$, V is equidistant from Y and Z. So, by the Converse of the Perpendicular Bisector Theorem, V is on the perpendicular bisector of $\overline{YZ}$, which is $\overleftrightarrow{WX}$.

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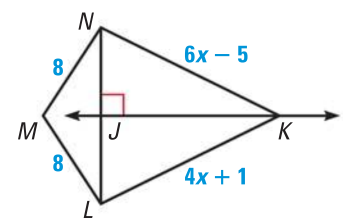
GUIDED PRACTICE for Examples 1 and 2

In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{NL}$.

1. What segment lengths are equal? *Explain* your reasoning.
2. Find NK.
3. *Explain* why M is on $\overleftrightarrow{JK}$.



ACTIVITY FOLD THE PERPENDICULAR BISECTORS OF A TRIANGLE

QUESTION Where do the perpendicular bisectors of a triangle meet?

Materials:
- paper
- scissors
- ruler

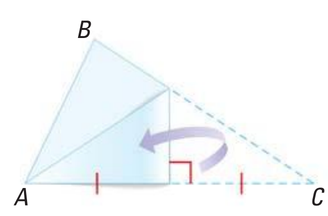
Follow the steps below and answer the questions about perpendicular bisectors of triangles.

STEP 1 Cut four large acute scalene triangles out of paper. Make each one different.

STEP 2 Choose one triangle. Fold it to form the perpendicular bisectors of the sides. Do the three bisectors intersect at the same point?

STEP 3 Repeat the process for the other three triangles. Make a conjecture about the perpendicular bisectors of a triangle.

STEP 4 Choose one triangle. Label the vertices A, B, and C. Label the point of intersection of the perpendicular bisectors as P. Measure $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. What do you observe?



CONCURRENCY When three or more lines, rays, or segments intersect in the same point, they are called **concurrent** lines, rays, or segments. The point of intersection of the lines, rays, or segments is called the **point of concurrency**.

READ VOCABULARY
The perpendicular bisector of a side of a triangle can be referred to as a *perpendicular bisector of the triangle*.

As you saw in the Activity on page 304, the three perpendicular bisectors of a triangle are concurrent and the point of concurrency has a special property.

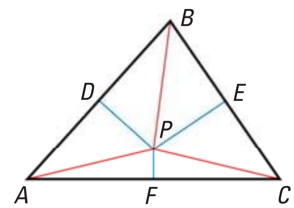
THEOREM *For Your Notebook*

THEOREM 5.4 Concurrency of Perpendicular Bisectors of a Triangle

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

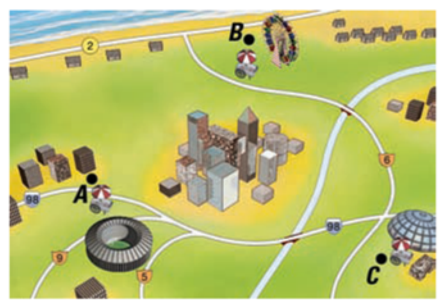
Proof: p. 933



EXAMPLE 3 Use the concurrency of perpendicular bisectors

FROZEN YOGURT Three snack carts sell frozen yogurt from points A, B, and C outside a city. Each of the three carts is the same distance from the frozen yogurt distributor.

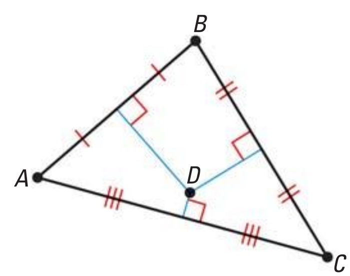
Find a location for the distributor that is equidistant from the three carts.



Solution

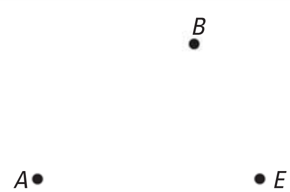
Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points A, B, and C and connect those points to draw $\triangle ABC$. Then use a ruler and protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency D is the location of the distributor.



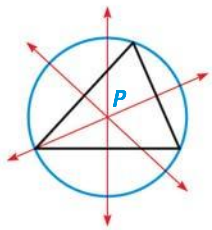
GUIDED PRACTICE for Example 3

4. **WHAT IF?** Hot pretzels are sold from points A and B and also from a cart at point E. Where could the pretzel distributor be located if it is equidistant from those three points? Sketch the triangle and show the location.

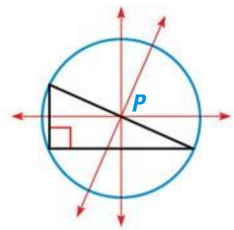


READ VOCABULARY
The prefix *circum-* means "around" or "about" as in *circumference* (distance around a circle).

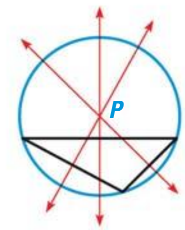
CIRCUMCENTER The point of concurrency of the three perpendicular bisectors of a triangle is called the **circumcenter** of the triangle. The circumcenter P is equidistant from the three vertices, so P is the center of a circle that passes through all three vertices.



Acute triangle
***P* is inside triangle.**



Right triangle
***P* is on triangle.**



Obtuse triangle
***P* is outside triangle.**

As shown above, the location of P depends on the type of triangle. The circle with the center P is said to be *circumscribed* about the triangle.

5.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 17, and 25
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 25, and 28

SKILL PRACTICE

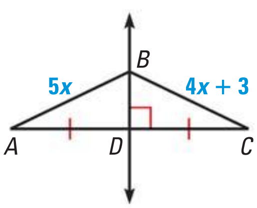
1. **VOCABULARY** Suppose you draw a circle with a compass. You choose three points on the circle to use as the vertices of a triangle. Copy and complete: The center of the circle is also the __?__ of the triangle.

2. ★ **WRITING** Consider $\overline{AB}$. How can you *describe* the set of all points in a plane that are equidistant from A and B?

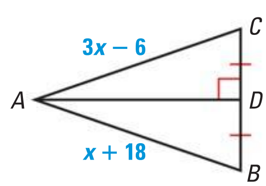
EXAMPLES 1 and 2 on pp. 303–304 for Exs. 3–15

ALGEBRA **Find the length of $\overline{AB}$.**

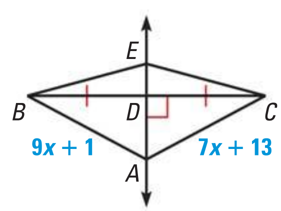
3.



4.

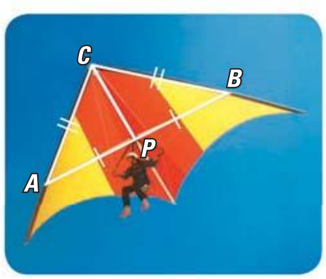


5.

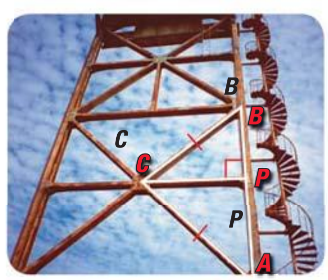


REASONING **Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of $\overline{AB}$.**

6.



7.



8.

9. ★ **MULTIPLE CHOICE** Point P is inside $\triangle ABC$ and is equidistant from points A and B. On which of the following segments must P be located?

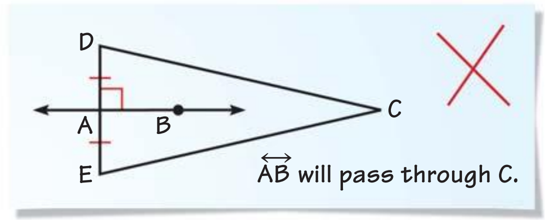
Ⓐ $\overline{AB}$

Ⓑ The perpendicular bisector of $\overline{AB}$

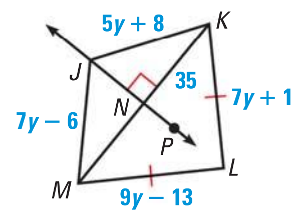
Ⓒ The midsegment opposite $\overline{AB}$

Ⓓ The perpendicular bisector of $\overline{AC}$

10. **ERROR ANALYSIS** *Explain* why the conclusion is not correct given the information in the diagram.



PERPENDICULAR BISECTORS **In Exercises 11–15, use the diagram. $\overleftrightarrow{JN}$ is the perpendicular bisector of $\overline{MK}$.**



11. Find NM.

12. Find JK.

13. Find KL.

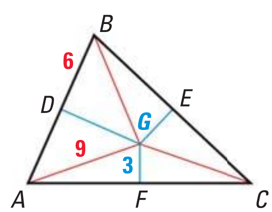
14. Find ML.

15. Is L on $\overleftrightarrow{JP}$? *Explain* your reasoning.

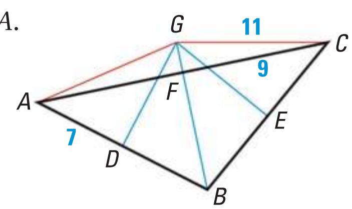
EXAMPLE 3
on p. 305
for Exs. 16–17

USING CONCURRENCY **In the diagram, the perpendicular bisectors of $\triangle ABC$ meet at point G and are shown in blue. Find the indicated measure.**

16. Find BG.



17. Find GA.



18. **CONSTRUCTING PERPENDICULAR BISECTORS** Use the construction shown on page 33 to construct the bisector of a segment. *Explain* why the bisector you constructed is actually the perpendicular bisector.

19. **CONSTRUCTION** Draw a right triangle. Use a compass and straightedge to find its circumcenter. Use a compass to draw the circumscribed circle.

ANALYZING STATEMENTS **Copy and complete the statement with *always*, *sometimes*, or *never*. *Justify* your answer.**

20. The circumcenter of a scalene triangle is __?__ inside the triangle.

21. If the perpendicular bisector of one side of a triangle goes through the opposite vertex, then the triangle is __?__ isosceles.

22. The perpendicular bisectors of a triangle intersect at a point that is __?__ equidistant from the midpoints of the sides of the triangle.

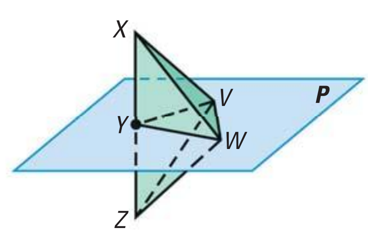
23. **CHALLENGE** Prove the statements in parts (a) – (c).

GIVEN ▶ Plane P is a perpendicular bisector of $\overline{XZ}$ at Y.

PROVE ▶ **a.** $\overline{XW} \cong \overline{ZW}$

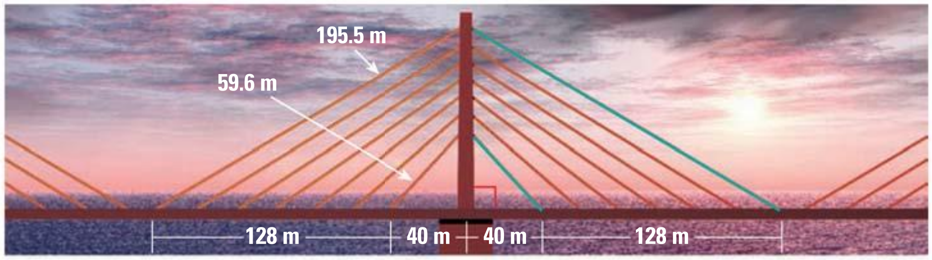
b. $\overline{XV} \cong \overline{ZV}$

c. $\angle VXW \cong \angle VZW$



PROBLEM SOLVING

24. **BRIDGE** A cable-stayed bridge is shown below. Two cable lengths are given. Find the lengths of the blue cables. *Justify* your answer.



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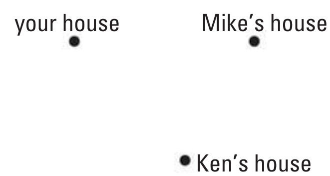
EXAMPLE 3 on p. 305 for Exs. 25, 28

25. ★ **SHORT RESPONSE** You and two friends plan to walk your dogs together. You want your meeting place to be the same distance from each person's house. *Explain* how you can use the diagram to locate the meeting place.



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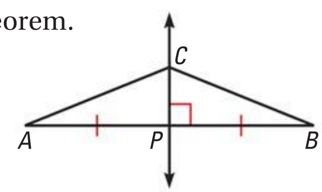
26. **PROVING THEOREM 5.2** Prove the Perpendicular Bisector Theorem.

GIVEN ▶ $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

PROVE ▶ $CA = CB$

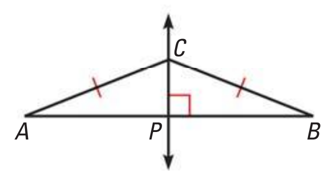
Plan for Proof Show that right triangles $\triangle APC$ and $\triangle BPC$ are congruent. Then show that $\overline{CA} \cong \overline{CB}$.



27. **PROVING THEOREM 5.3** Prove the converse of Theorem 5.2. (*Hint:* Construct a line through C perpendicular to $\overline{AB}$.)

GIVEN ▶ $CA = CB$

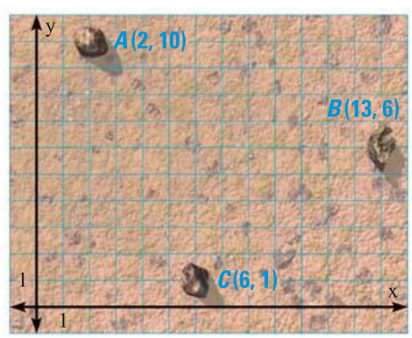
PROVE ▶ C is on the perpendicular bisector of $\overline{AB}$.



28. ★ **EXTENDED RESPONSE** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community firepit at its center. They mark the locations of Stones A, B, and C on a graph where distances are measured in feet.

   a. *Explain* how the archaeologists can use a sketch to estimate the center of the circle of stones.

   b. Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the firepit.



29. **TECHNOLOGY** Use geometry drawing software to construct $\overline{AB}$. Find the midpoint C. Draw the perpendicular bisector of $\overline{AB}$ through C. Construct a point D along the perpendicular bisector and measure $\overline{DA}$ and $\overline{DB}$. Move D along the perpendicular bisector. What theorem does this construction demonstrate?

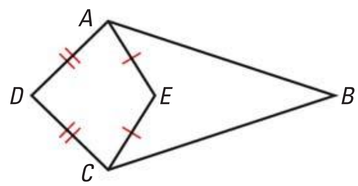
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

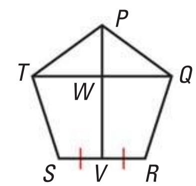
30. **COORDINATE PROOF** Where is the circumcenter located in any right triangle? Write a coordinate proof of this result.

PROOF **Use the information in the diagram to prove the given statement.**

31. $\overline{AB} \cong \overline{BC}$ if and only if D, E, and B are collinear.



32. $\overline{PV}$ is the perpendicular bisector of $\overline{TQ}$ for regular polygon $PQRST$.



33. **CHALLENGE** The four towns on the map are building a common high school. They have agreed that the school should be an equal distance from each of the four towns. Is there a single point where they could agree to build the school? If so, find it. If not, *explain* why not. Use a diagram to *explain* your answer.

MIXED REVIEW

Solve the equation. Write your answer in simplest radical form. *(p. 882)*

34. $5^2 + x^2 = 13^2$

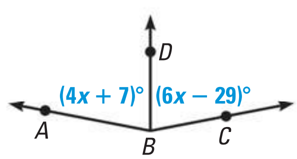
35. $x^2 + 15^2 = 17^2$

36. $x^2 + 10 = 38$

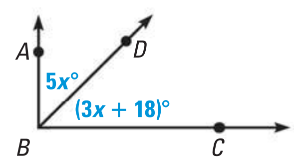
PREVIEW
Prepare for Lesson 5.3 in Exs. 37–38.

Ray $\overrightarrow{BD}$ bisects $\angle ABC$. Find the value of x. Then find $m\angle ABC$. *(p. 24)*

37.



38.



Describe **the pattern in the numbers. Write the next number.** *(p. 72)*

39. 21, 16, 11, 6, . . .

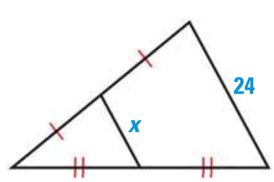
40. 2, 6, 18, 54, . . .

41. 3, 3, 4, 6, . . .

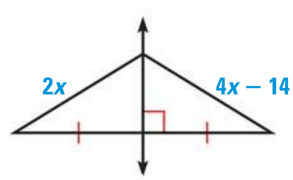
QUIZ *for Lessons 5.1–5.2*

Find the value of x. Identify the theorem used to find the answer. *(pp. 295, 303)*

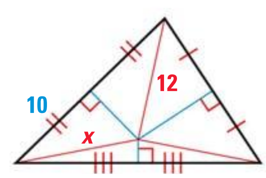
1.



2.



3.



4. Graph the triangle $R(2a, 0)$, $S(0, 2b)$, $T(2a, 2b)$, where a and b are positive. Find RT and ST. Then find the slope of $\overline{SR}$ and the coordinates of the midpoint of $\overline{SR}$. *(p. 295)*